Advanced Algorithm

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Lecture 5: Permutation Routing Problem

Permutation Routing Problem

- Randomized Algorithm - Chapter 4.2, P74
- Routing Problem: Given graph $G = (V, E)$, every node i wants to send one packet to some other node $d(i)$. Every round, every edge direction can transfer at most one packet. How many rounds?
- Permutation Routing Problem: $\{d(i)\}$ is a permutation.
- Oblivious algorithm: the route from node i to $d(i)$ depends on $(i, d(i))$ only.

Theorem

For any deterministic oblivious permutation routing algorithm on a graph of N nodes each of out-degree d, there is an instance of permutation routing requiring $\Omega(\sqrt{N/d})$ steps.

- Ref: "Routing, merging and sorting on parallel models of computation" by A. Borodin, J.E. Hopcroft. (1985)

Permutation Routing Problem

We only consider n-dimension Boolean hypercube in this lecture. $N = 2^n$ nodes.

1. First attempt: deterministic algorithm
   - Bit fixing algorithm;
   - Consider permutation: $(x, y) \to (y, x)$ where x, y are $\frac{n}{2}$-bit strings.
2. Second attempt: randomized bit fixing algorithm
3. Third attempt: two-phase randomized bit fixing algorithm, by L. Valiant(1981)

Homework

1. In randomized bit fixing algorithm, consider the permutation $(x, y) \to (y, x)$ where x, y are $\frac{n}{2}$-bit strings. Prove its running time is $2^{\Omega(n)}$ with high probability $(1 - o(1))$.
2. In Valiant two-phase algorithm, prove $Pr(\exists x, delay(x) > cn) = o(1)$.

Lecture 5.2: Probabilistic Method

- Ref. Probabilistic Method - Chapter 1
- Ramsey number $R(m, n)$: smallest number k such that in any 2-coloring of the edges of a complete graph on k vertices by red and blue, there either is a red K_n or a blue K_m. We are interested in the lower bound of $R(m, n)$.
- $R(2, 2) = 2, R(3, 3) = 6$.
- $R(4, 5) = 25$ (1995), $43 \leq R(5, 5) \leq 49$.
- Constructive proof: $R(m, n) \geq (m - 1)(n - 1) + 1$.
- (Paul Erdös) $R(n, n) \geq 2^{\frac{n}{2}}$

Probabilistic Method

- Goal: we want to prove the existence of some kind of structure.
- We construct some kind of random process.
- Prove the probability of the event that this structure exists is strictly greater than 0.

Max Cut problem

- Ref. Randomized Algorithm - Chapter 5.1, Page 103.
- NP-hard problem
- For any undirected graph $G(V, E)$ with n vertices and m edges, there is a partition of the vertex set V into two sets A and B such that $|\{(u, v) \in E \mid u \in A, v \in B\}| \geq m/2$.
  - Proof?
  - Construction?

Max-SAT problem

- Ref. Randomized Algorithm - Chapter 5.1, Page 103.
- NP-hard problem
- For any set of m clauses, there is a truth assignment for the variables that satisfies at least $m/2$ clauses.
  - Proof?
  - Construction?

Homework

Randomized Algorithm - Problem 5.3, Page 124